

Endogenous agency problems and the dynamics of rents

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Abstract

Agents choose to acquire skills ranging from simple and transparent tasks to complex and opaque ones. While potentially more productive, the latter generate more severe agency problems. In our overlapping generations model, agents compete with their predecessors. With dynamic contracts, long horizons help principals incentivize agents. Agents with short horizons are more difficult to incentivize than agents with long horizons. Hence, old agents are imperfect substitutes for young ones. This reduces competition between generations. As a result, young managers can opt for more opaque and complex technologies, and therefore larger rents, than their predecessors. Thus, in equilibrium, complexity and rents rise over time. Our theoretical results are in line with the increase in complexity and rents observed in the finance sector.

Keywords: Agency rents, moral hazard, finance sector, dynamic contracts, opacity.

JEL codes: D8, G2, G3.

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1 Introduction

Agency problems arise when principals cannot precisely observe and control what agents do and how they use resources. In this context, to provide agents with the right incentives, principals must leave them rents. The severity of the agency problem, and the magnitude of the corresponding rents, depend on the characteristics of the task delegated to the agent. This paper offers an equilibrium analysis of the endogenous determination of these characteristics, and the corresponding dynamics of rents.

In Holmstrom (1979), Grossman and Hart (1983), and Holmstrom and Tirole (1997), the agent's ex-ante choice between effort and shirking is unobservable. To provide incentives to the agent, the principal must promise her large compensation in case of success, which gives rise to rents. As shown in Holmstrom and Tirole (1997), these rents increase with the cost of effort, or, equivalently, the private benefit from shirking. Now, the more complex the task, the larger the effort the agent must put in to master the difficulty and achieve success, i.e., the larger the cost of effort, and hence the agency rents. Moreover, the greater the opacity of the task, the more difficult it is for the principal to monitor and control the agent.² Thus, opacity enhances the ability of the agent to engage in actions and use resources that benefit him rather than the principal.³ This corresponds to large private benefits from shirking, and hence large rents.

Thomas and Worall (1988), Kocherlakota (1998), Townsend (1979), Diamond (1984), and Bolton and Scharfstein (1990), emphasize ex-post unobservability and limited commitment. A problem arises when the agent cannot commit not to leave the firm, while it would be very difficult to complete the task successfully if the agent absconded. In this context, the principal must leave the agent a rent, to convince him not to abscond. The more complex and opaque the task, the more difficult it is to complete it without the agent, the larger the rent. A particularly perverse example, was offered by AIG, where, after very large losses, the principals (in fact the taxpayers) had to pay large compensation to retain managers. As written in the new York Times (March 16, 2009): "AIG employees concocted complex derivatives ... taxpayers need to keep some of these brainiacs in their seats.. only they can

²As shown in Sato (2015), opacity increases information asymmetry between the principal and the agent.

³Similarly, complexity can reduce the ability of the principal to monitor the agent. This can arise due to bounded rationality, in line with the analysis of Brunnermeier and Oehmke (2008): because of information overload, boundedly rational principals cannot check each step, nor grasp the entirety, of complex processes. Furthermore complex tasks are less standard, and therefore more difficult to benchmark.

... figure out how to unwind these tricky investments.” While extreme, this case vividly illustrates the link between complexity and rents.

Now, the literature often takes as given the characteristics of the agent’s task, and of the corresponding agency problems. Yet, in practice, agents acquire skills appropriate for activities, sectors or products with various levels of complexity and opacity. Then firms hiring these agents engage in activities with agency problems of various magnitudes.⁴ This paper studies the equilibrium choice of skills and tasks, and of the corresponding dynamics of agency problems and rents.

To clarify the origin of rents in our analysis, we assume there is no scarcity of managers. Thus, if the market for managers was frictionless, principals would hire only those managers that are optimal from their point of view, maximizing returns net of rents. Since agents would choose to acquire only those skills that make them employable, complexity and opacity would not rise above what is optimal for principals. In contrast, we assume there are frictions in the labour market.

We consider an overlapping generations model, in which agents live two periods. At the beginning of his life, a generation t young manager chooses (at a cost) a given skill and a corresponding task, denoted by b . b parametrizes both the productivity of the agent and the magnitude of the agency problem. Large b s can correspond to more productive activities, but they also entail larger agency rents. Then, each young principal meets a young agent, observes his b , and decides whether to hire him or not. When making this choice, the principal bears in mind that she could instead *i*) search for another generation t agent or *ii*) hire a generation $t - 1$ agent, and then another agent at time $t + 1$. We assume principals incur a (possibly very small) cost when searching managers. This shuts down competition between contemporaneous managers, as in Diamond (1971), enabling one to focus on the key driving force in our model: competition between successive generations.

It is particularly attractive for a generation t principal to try and hire a generation $t - 1$ agent if low b s were chosen by that generation. Thus, when generation $t - 1$ chose relatively transparent and simple tasks, this limits the ability of generation t to increase its own b to earn high rents. The competitive pressure imposed by the previous generation is limited, however, by the *endogenously* imperfect substitutability among generations. The intuition is the following: To reduce rents, principals

⁴Again, AIG offers an extreme, but vivid, illustration: Traditionally the firm offered standard insurance services. Then, starting, in the early 2000s, AIG started hiring highly skilled financial engineers and dealing in complex credit default derivatives, for which agency problems were much more severe.

defer compensation (as in Becker and Stigler, 1974, and Rogerson, 1985). This makes it relatively unattractive for a young principal to hire an old agent. Indeed, old agents have short horizons, which prevents deferring their compensation to reduce their rents. Thus, other things equal, it is cheaper to incentivize young agents than old ones. Because old agents are imperfect substitutes for young ones, the latter can afford to choose technologies with greater agency problems than their predecessors, and still be hired. This gives rise to an upward trend in complexity, opacity, incentive problems and rents. This progressive deterioration in transparency and simplicity standards, paralleled by an increase in rents, is the core result of our paper. Note that, even when higher complexity raises *gross* returns, it eventually results in lower *net* returns for investors.

Since, when choosing its b , generation $t - 1$ sets a benchmark for generation t , while the actions of generation $t - 1$ have no *exogenous direct* effect on the following generation, they exert an *endogenous externality* on the latter. Now, when choosing a relatively high level of complexity, generation $t - 1$ does not internalize that this will lead to an even larger level of complexity at the next generation. Consequently, equilibrium can be inefficient, calling for public policy intervention. To analyze this point, we consider a regulator, who can set transparency and simplicity standards, monitor agents, and punish those who don't comply, i.e., whose b s are above the standard.

First, we analyze the permanent monitoring case, in which the regulator sets a constant transparency and simplicity benchmark. In this case, we show that multiple equilibria can arise, in contrast with the *laissez faire* case, where equilibrium is unique. Under *laissez faire*, generation t managers are disciplined by their competitors from the previous generation. Thus, the equilibrium choice of b at time t is the unique best response to the predetermined level of b at time $t - 1$. In contrast, with permanent monitoring, the choice faced by a generation t manager depends on the anticipated choice of his contemporaneous competitors. If the latter are expected to comply with tough standards, then the generation t manager prefers to do the same, to remain employable. If, on the contrary, it is expected that managers don't comply, and deviate to high b s, then each generation t manager can find it optimal to deviate. This can be interpreted in terms of social norms: When the norm is compliance, i.e., everyone is expected to comply, then the optimal action is to stick to this "ethical" norm. When the norm is non-compliance, i.e., everyone is expected to deviate to high b s, above the standard requested by the regulator, it can be individually optimal to deviate. Thus, when norms are

“unethical” (and “all continue to dance as long as the music is playing”), light-touch regulation is ineffective. In this context, to ensure compliance, the regulator must go for very intense (and very costly) monitoring.

Second, we analyze periodic monitoring, and show that it can dominate such costly permanent monitoring. With periodic monitoring, the regulator, when it intervenes, chooses a very high monitoring probability. This ensures compliance. After that, the regulator stops monitoring. There is, consequently, an upward trend in complexity and rents. But that trend is limited, because each generation sets a benchmark disciplining its successors. After some time, however, *bs* have grown a lot, standards have declined, and it is optimal that the regulator, again, intensively monitors for one period.

Empirical implications: While the economic mechanisms we analyze can be at play in various settings, our assumptions are particularly relevant for the financial sector. First, innovative financial activities are often complex and difficult to understand for outside investors (as illustrated by the above mentioned AIG case). This induces asymmetric information about the distribution of returns. Opacity increases this information asymmetry. For example, private equity or hedge fund managers, whose strategies are not explained in detail to investors, have private information about the distribution of the returns of their funds.⁵ Second, finance faces much less hard-wired technological constraints than manufacturing. In other words there is more plasticity or flexibility in financial activities. As written by Henderson and Pearson (2011), “the ability to create instruments with almost any payoffs implies that there are few limits on the complexity of financial instruments.” This opens up the scope for creativity. Third, accidents caused by malfunctioning financial innovations are less life threatening than those caused by innovations in other sectors, e.g., medicine or public transportations. Hence, innovation is less regulated and monitored in finance. Again, this opens up the scope for creativity. Our theoretical analysis shows how finance sector managers can take advantage of these characteristics of the sector to earn rents.

Indeed, the implications of our theory are in line with empirical findings about the financial sector. Our model generates a simultaneous increase in rents and complexity, which is consistent with the

⁵This can give rise to the “fake alpha” problem highlighted by Rajan (2008).

findings of Philippon and Resheff (2008). They find that, while in 1980, managers' earnings were similar in the finance sector and other sectors, in 2006, finance managers earned 70% more than comparable managers in other sectors.⁶ Philippon and Resheff (2008) also document that, during this period, the complexity of the tasks and occupations of finance sector managers grew very significantly. Their analysis emphasizes the role played by deregulation in this context. Our theoretical result that market forces may not prevent the rise of rents and opacity, while regulation could stem it, is in line with their findings. While Philippon and Resheff (2008) document the growth in the complexity of the tasks of finance managers, Greenwood and Sharfstein (2012) document an increase in the complexity and opacity of the products and techniques in that sector. They find that as the finance sector was growing, the share of private equity and hedge funds increased relative to the share of standard mutual fund management. This led to an increase in overall complexity and opacity, since hedge funds and private equity funds are more opaque and complex than standard mutual funds. Another manifestation of the increase in complexity and opacity is the growth, documented by Greenwood and Sharfstein (2012), of the shadow banking system. As noted by Greenwood and Sharfstein (2012) shadow banking lengthens the credit intermediation process. This, in turn, increases opacity. Anecdotal evidence also points in the same direction. For example, Greg Smith, an executive director at Goldman Sachs, decided to resign after 12 years in that institution. In the article he wrote on that topic in *The New York Times* on March 14, 2012, he underscored the rise of complexity, i.e., the increasing tendency to “pitch lucrative and complicated products to clients even if they are not the simplest investments or the ones most directly aligned with the client’s goals.”⁷

Our theory predicts that younger generations of managers tend to use more complex or opaque financial products than older generations. This echoes findings in the empirical literature: Almazan et al. (2004) look at restrictions found in the contracts between mutual fund investors and managers on complex investment techniques such as shorting or using options. Consistent with the implications

⁶Additional evidence about the finance premium is offered by Goldin and Katz (2008), who find that, in 2005, Harvard graduates working in finance earned 95% more than those working in other sectors. This is also consistent with anecdotal evidence, e.g., an article entitled “Bank staff costs take bigger share of pot” (Financial Times, June 5, 2012) stated that, in a survey of 13 global financial institutions in 2011, total bankers' compensation was 81% of the sum of dividends, retained earnings and bankers' pay.

⁷See http://www.nytimes.com/2012/03/14/opinion/why-i-am-leaving-goldman-sachs.html?_r=1&src=me&ref=general

of our model, they find that restrictions decline over time (their time-period is 1994-2000) and that newer funds and younger managers tend to be significantly less constrained than older ones.⁸

Another implication of our theoretical analysis is that the increase in rents, opacity and complexity spurred by an initial deregulation shock is not instantaneous. Rather it can be sustained and delayed. Hence our theory implies that sustained increases in rents and opacity can take place at points in time where there is no current changes in exogenous variables. This is a caution against empirical analysis attempting to match changes in rents and opacity with contemporaneous or recent changes in regulation.

Yet another implication of our analysis relates the increase in rents to the search for yields. In general, the rise of opacity and rents is limited by the constraint that agents must leave enough return to the principals to convince them to delegate the management of their wealth rather than self-invest it. When the safe return and the return on indexing are low, so is the return on self-investment. This increases the ability of agents to increase opacity to extract rents.

Finally, our analysis implies that experienced managers and junior managers are imperfect substitutes. This should show up in hiring and compensation data. For example, when new slots open up, experienced managers are imperfect substitutes for junior ones. Our theory also predicts that imperfect substitutability, and its consequences, should be stronger when incentive problems are more severe and when compensation is more backloaded.

Literature: Our analysis of the welfare costs induced by agents' opting for socially unproductive rent-seeking is in line with Baumol (1990) and Murphy, Shleifer and Vishny (1991). Both in their analysis and ours, rent-seeking agents impose costs upon the others. But, in Baumol (1990) and Murphy, Shleifer and Vishny (1991) these costs are directly induced by the actions of the rent-seeker, e.g., warfare, litigation or predatory trading. In contrast, in our analysis, the initial choice of the agent (the level of opacity and complexity of the activity) has an indirect endogenous impact on the principal, via the agency rent it induces, and also on subsequent increases in complexity.

⁸Similarly, Chernenko et al. (2014) find that managers who are younger or less experienced were more likely to buy non-traditional securitized products (mortgage backed securities backed by nonprime loans) prior to the 2007 financial crisis.

Our work is also related to Myerson (2012)'s and Axelson and Bond (2015)'s equilibrium analyses of dynamic contracting with overlapping generations and moral hazard. An intriguing feature of the model in Axelson and Bond (2015) is that agents can be assigned to two types of task, with different levels of moral hazard and productivity. Thus, a common theme in their paper and ours is the choice of tasks, and corresponding moral hazard, arising in equilibrium. The endogenous rise in rents, reflecting imperfect competition between successive generation, is one of the key specific results of our model, differentiating it from Myerson (2012) and Axelson and Bond (2015).

Our point that agents in the finance industry choose complex products and techniques to increase the rents they extract from principals, echoes the point made by Carlin (2009) that competing financial institutions design complex products to increase their market power. A major difference is that our analysis hinges on agency problems, which can arise even with large rational investors, while Carlin (2009) focuses on retail investors and abstracts from agency issues.

Our analysis is also related to that of Bolton, Santos and Scheinkman (2013). In their paper also, opportunistic occupational decisions lead to rents, opacity and inefficiencies. Also, both papers uncover externalities associated with the development of opaque activities and markets. The economic mechanisms at work in the two papers are different, however. In Bolton et al (2013), when many agents choose to become dealers in the opaque OTC market, this worsens *adverse selection* in the other (transparent) market, which increases the bargaining power, and hence the rents, of *contemporaneous* OTC dealers. In contrast, in our analysis, when agents choose opaque investment techniques, this worsens *moral hazard*, and increases rents, for the *following generations*.

Last, our paper is related to the literature on social norms, which explores, notably, how parents transmit values or preferences to their children (see, e.g., Bisin and Verdier, 2000). In our model, in contrast with that literature, the transmission of norms from one generation to the next is driven by competition between generations. And we show that the imperfection of that competitive process induces a decline in standards.

The next section presents the model. Section 3 presents the the optimal contract designed by one principal, hiring one agent for two periods. Section 4 embeds this bilateral contracting problem in an equilibrium labour market context and analyzes the dynamics of rents. Section 5 discusses welfare and policy. Section 6 briefly concludes.

2 Model

Investors and managers: Consider an overlapping generations model, where, each period, a mass-one continuum of investors and a mass- M continuum of managers are born. $M \geq 1$, so that there is no scarcity of managers. Also, because we consider an overlapping generations model, successive generations of managers coexist in the market at a given point in time, which creates the scope for competition between generations.

All market participants are risk neutral, have limited liability and live for two periods. The discount factor of investors is $\rho \in (0, 1)$, while that of managers is $\beta \in (0, 1)$. In line with the literature on dynamic financial contracting (e.g., DeMarzo and Fishman (2007) and Biais, Mariotti, Plantin and Rochet (2007)), we assume $\rho \geq \beta$.

Each investor is initially endowed with one unit of investment good. She can invest it in a default technology, which she can operate herself and which returns 1 unit of consumption good per period during two periods. Alternatively, she can delegate the management of her capital to an agent, hereafter referred to as “the manager”. For simplicity the choice between self-investment and delegated investment is irreversible.

Managers have zero initial endowment. At the beginning of his life, each young manager must choose among a range of investment techniques indexed by $b \in [0, 1]$. Each technique corresponds to a specific type of skills, knowhow and human capital. The (non-monetary) cost of acquiring skills b is equal to cb , with $c \geq 0$. b indexes the sophistication and complexity of the investment technique. c is the cost of acquiring the skills necessary to design complex products and strategies.

Importantly, the choice of b is irreversible. The idea is that managers acquire skills, human capital, relations and knowledge of investment techniques at an early stage in their career. Then, they use this informational capital.⁹ We hereafter often refer to the b chosen by a manager as his “type.”

When entrusted with one unit of capital, a manager with sophistication b can generate return equal to $R(b) \geq 1$ units of consumption good per period during each of the two periods of his life. We assume R is continuous, increasing and concave in the sophistication of the investment, b .¹⁰ $R'(b)$

⁹See Oyer (2008) for empirical evidence on long term effects of initial career paths in the financial sector.

¹⁰The assumption that R is increasing is not needed for our analysis. All our qualitative results are upheld when R is constant. Moreover, one can interpret $R(b)$ as the efficient frontier of the production set, in b /output plane.

denotes the left derivative of $R(b)$. Since R is concave, R' is decreasing and, in the same as spirit as Inada conditions, we assume that $R'(1) = 0$.

Agency problems: While sophistication can increase gross returns, it also increases the complexity of the investment process, its opacity for investors, and, thus agency problems. To model these problems, we assume, as in Holmstrom and Tirole (1997), that at the beginning of each period, the agent can exert effort or shirk. One can equivalently assume that effort is costly or that shirking generates private benefits.¹¹ The cost of effort includes the cost of screening and selecting the right projects, carefully implementing them, and ensuring they are completed. This cost increases with the complexity of the task. Private benefits arise when agents engage in activities that are beneficial for them, without increasing the returns for the principal. This includes activities enabling the agent to acquire reputational, human or relational capital. Opportunities for private benefits extraction increase with complexity and opacity of the task. Following Holmstrom and Tirole (1997) we cast our model in terms of private benefits from shirking.

For simplicity, we assume that, when the agent exerts effort, the project generates cash flow $R(b)$ for sure, while, when the agent fails to exert effort, cash flows can be $R(b)$ with probability $1 - \Delta$, or 0 with probability Δ . The agency problem arises because only the agent knows if he exerted effort or shirked (effort is unobservable by investors) and the agent has limited liability. We assume the private benefit from shirking is equal to $b\Delta R(b)$. This can be interpreted as follows: Private benefits from shirking are equal to a fraction ($b \in [0, 1]$) of the loss in expected output due to shirking ($\Delta R(b)$), so that (other things equal) increasing b raises the attractiveness of shirking for the agent. Thus, we consider a set of investments technologies, indexed by b , varying in terms of productivity and agency problems, and we interpret tasks that are potentially more productive, but also exposed to more severe agency problems, as complex and opaque.¹²

Sequence of play: Within each period $t \geq 1$, the timing of actions is the following:

¹¹The difference is that, while the cost of effort is incurred on the equilibrium path, the private benefits from shirking are not. This leads to slightly simpler expressions.

¹²For simplicity and brevity, we don't model explicitly how complexity and opacity affect agency problems. For a model of that link see Sato (2015).

- Stage 1: Young manager i in generation t chooses $b_t^i \in [0, 1)$.
- Stage 2: Each young investor is matched with one young manager, observes his b_t^i , and decides whether to make him a take-it-or-leave-it contract offer or reject him.¹³ Since there is a mass one of investors, and a mass $M > 1$ of managers, each manager is matched with an investor with probability $1/M$. This probability is the same for all managers. In particular, it cannot depend on the choices made by managers at stage 1, because an investor observes a manager's b only after being matched with him.
- Stage 3: If the investor decides to reject the young manager with whom she was matched, or if the manager rejects the offer, then the investor decides whether to self-invest or search for another manager, at cost ϵ , which can be arbitrarily small. If the investor decides to continue searching for managers, she can direct her search towards young or old managers. Then, on meeting a new manager, the investor observes his type and can make him a take-it-or-leave-it offer, and the process is iterated. Eventually, investment takes place.
- Stage 4: Each employed manager decides whether to exert effort or not, then output is realized (and equals $R(b_t^i)$ or 0), and the manager receives the compensation stated in the contract.

This timeline is illustrated in Figure 1. At period $t + 1$, the investor and the manager continue to apply contract signed at stage 2 of period t . If the time t output was 0 and the contract stated the manager should be fired in that case, the investor can search for a new agent, at cost ϵ .

3 Optimal contracting

In this section, we analyse the optimal contract designed by one principal, hiring one agent, taking b_t as given. In the next section we embed this contracting problem in a market equilibrium and study the endogenous determination of b_t . The compensation contract offered at time t by the investor states the wages to be received by the manager as a function of the output realized at time t and at time $t + 1$. It also specifies if the manager should be kept after period t or fired.

¹³For simplicity we assume principals can commit to the contracts they offered to an agent.

First consider the contract requesting the agent to exert effort at both times. In that case, on the equilibrium path, the output is equal to $R(b_t)$ at each period. It is clearly optimal to fire the manager, without any compensation, when output is 0. Hence, the time t contract is pinned down by the pair of wages, w_t^t and w_{t+1}^t , paid to the manager if output $R(b_t)$ is generated in period t and in period $t + 1$. After success at time t , the incentive compatibility condition at time $t + 1$ is that the gain of the agent when exerting effort (wage w_{t+1}) be larger than or equal to his gain when shirking (wage with probability $1 - \Delta$ plus private benefit from shirking)

$$w_{t+1}^t \geq (1 - \Delta)w_{t+1}^t + \Delta b_t R(b_t),$$

that is

$$w_{t+1}^t \geq b_t R(b_t). \quad (1)$$

At the end of period t , after $R(b_t)$ has been obtained, the continuation utility of the agent, anticipating effort at $t + 1$, is βw_{t+1} . Thus, at time t the incentive compatibility condition is

$$w_t + \beta w_{t+1}^t \geq (1 - \Delta)(w_t^t + \beta w_{t+1}^t) + \Delta b_t R(b_t).$$

That is

$$w_t^t + \beta w_{t+1}^t \geq b_t R(b_t). \quad (2)$$

In this section we normalize the outside option of the manager to 0, so that his participation constraint never binds. In the next section, the endogenous outside option of the manager will still be 0. The program of the investor is to maximize expected net returns subject to incentive compatibility constraints, i.e.,

$$\max_{w_t^t, w_{t+1}^t} R(b_t)(1 + \rho) - w_t^t - \rho w_{t+1}^t, \text{ s.t., (1) and (2)}. \quad (3)$$

The solution to this program is spelled out in the next lemma.

Lemma 1: *At time t , for a given choice of b_t , the solution to (3) is such that (1) and (2) bind and the wages conditional on success are*

$$\{w_t^t, w_{t+1}^t\} = \{(1 - \beta)b_t R(b_t), b_t R(b_t)\}.$$

Complexity, net returns and rents. By Lemma 1, the present value of the fund manager's earnings is

$$w_t^t + \beta w_{t+1}^t = b_t R(b_t). \quad (4)$$

Thus, the agent captures a fraction (b_t) of the gross return on the investment over one period ($R(b_t)$). As the complexity of the investment technique (b_t) increases, the total gross return increases, because $R' \geq 0$. In addition, the fraction of that return captured by the agent also increases, because the agency problem worsens. Combining these two effects, the compensation of the agent increases with the complexity of the investment technique.

While agents benefit from an increase in complexity, principals can be made better off or worse off when complexity rises. Indeed,

$$Z'(b_t) = [(1 + \rho) - (1 + \rho - \beta)b_t]R'(b_t) - (1 + \rho - \beta)R(b_t).$$

Because $(1 + \rho) > (1 + \rho - \beta)b_t$ and $R'(b)$ is decreasing, the investors' net return is concave in the complexity of the investment technique. And because $R'(1) = 0$, we have $Z'(1) \leq 0$. Thus, starting from $Z(0)$, investors' net return initially increases with b , reflecting the increase in gross return $R(b)$. Then, it reaches a maximum point at

$$b^* = \arg \max_b Z(b_t). \quad (5)$$

Finally, for $b > b^*$, investors' net return goes down with b , reflecting that an increasing fraction of the return is captured by the agent. To make things interesting, we assume $Z(0) \geq 1 + \rho$, i.e., at the lowest level of complexity the enhancement in net return brought about by an increase in b exceeds the cost c . Finally denote by b_{\max} the highest value of b in $[0, 1]$ such that $Z(b) \geq 1 + \rho$, i.e., investors prefer delegated investment rather than self-investment.

Optimality of effort: For the contract in Lemma 1 to be the optimal contract, it must generate higher net gains for the principal than the alternative contracts requesting i) no effort at all, or ii) effort at time t and no effort at time $t + 1$, or iii) no effort at time t and effort at time $t + 1$. By Lemma 1, the net gains of the investor requesting effort at both periods are

$$Z(b_t) \equiv R(b_t)[(1 + \rho) - (1 + \rho - \beta)b_t]. \quad (6)$$

On the other hand, if the principal lets the agent shirk at both periods, he does not need to pay any wage, and his net gains are

$$R(b_t)(1 + \rho)(1 - \Delta). \quad (7)$$

(6) is greater than (7) iff

$$\Delta \frac{1 + \rho}{1 + \rho - \beta} \geq b_t. \quad (8)$$

Since $b_t \in [0, 1]$, this always holds iff Δ is large enough, in the sense that

$$\Delta \geq \frac{1 + \rho - \beta}{1 + \rho}. \quad (9)$$

As shown in the appendix, under (9) shirking once is also dominated by effort at both periods. Hence, the contract spelled out in Lemma 1 is the optimal contract if (9) holds, which, for simplicity, we assume hereafter. If we did not make that assumption, there would be a threshold value of b at which the principal would prefer to give up on effort. In equilibrium, agents would not set b above that threshold, whose role would be similar to that of b_{\max} . Apart from that, relaxing (9) would not alter our results.

Example: A simple example is when $R(b)$ is the piecewise linear function $\min[\alpha b + 1, \bar{R}]$, where α is a positive constant. As explained in the appendix, for this simple case, if the agency problem is not too severe, in the sense that

$$\frac{1 + \rho}{1 + \rho - \beta} \geq \frac{2\bar{R} - 1}{\alpha}, \quad (10)$$

then $Z(b)$ is increasing for $b \leq \frac{\bar{R}-1}{\alpha}$, and $b^* = \frac{\bar{R}-1}{\alpha}$. This example is illustrated in Figure 2.

4 Equilibrium dynamics

We now turn to the dynamics of the choice of investment techniques by managers and the corresponding equilibrium outcomes. We focus on symmetric equilibria, in which all managers born at time t choose the same equilibrium level of complexity, b_t^* .

As stated in (4), the present value of an employed manager's earnings is $bR(b)$ which is increasing in b . For simplicity, we assume that the value function of an employed manager, $bR(b) - cb$, is also

increasing in b .¹⁴ This implies that, at any time t , $b_t < b^*$ would be Pareto dominated since both managers and investors would be better off with a larger b . So we initialize the process at $b_0^* = b^*$. From that point on, any increase in complexity reduces the net returns of investors, while raising the rents of managers. Thus, there is a conflict of interest between the former and the latter. We now study whether market forces lead to an equilibrium that is more favourable for the investors (keeping complexity at b^*) or for the managers (letting complexity rise above b^*).

Given an initial level of complexity, b_0^* , an equilibrium is a sequence $\mathcal{E} = \{b_t^*, w_t^{t*}, w_{t+1}^{t*}\}_{t \geq 1}$, satisfying the following conditions:

- **Optimization:** At each time t , each young manager i chooses b_t^i to maximize his gains, and each investor makes an optimal hiring decision.
- **Rational expectations:** Investors and managers have rational expectations about the equilibrium dynamics \mathcal{E} and find it optimal to also play according to \mathcal{E} . Thus, on the equilibrium path at time t , young manager i finds it optimal to set $b_t^i = b_t^*$, and each investor offers the optimal contract

$$\{w_t^{t*}, w_{t+1}^{t*}\} = \{(1 - \beta)b_t^* R(b_t^*), b_t^* R(b_t^*)\}. \quad (11)$$

In each generation, at stage 2, each manager is drawn with probability $\frac{1}{M}$. This probability does not vary with managers' types, because we assume that, before contacting the manager, the investor cannot observe the manager's type. Once drawn, a manager strictly prefers to be hired and earn (4). Thus, at stage 1, manager i chooses b_t^i to maximize his expected gains

$$\frac{1}{M} R(b_t^i) b_t^i - c b_t^i, \quad (12)$$

subject to the constraint that the investor be willing to hire him when drawing him, and the participation constraint that his ex-ante expected profit be non-negative. Since $b_t^i \geq b^*$, the participation constraint holds under the following maintained assumption

$$\frac{R(b^*)}{M} \geq c. \quad (13)$$

¹⁴More formally, the condition is $\frac{1}{M}(R(b) + bR'(b)) \geq c, \forall b$.

(13) implies that the rents which must be left to the agent to ensure incentive compatibility are always large enough that the agent recoups from the initial cost of acquiring skills. If (13) did not hold, since principals cannot commit to compensation before they meet agents, there would be a market breakdown: Agents would not acquire skills ex-ante, because they would anticipate not to be adequately rewarded for these ex-post.¹⁵

We now analyze the condition under which the investor prefers to hire agent i after drawing him from the pool, rather than opting for self investment, or continuing to search for managers. To do so, we need to compare the investor's payoff when hiring the young manager to her payoff from alternative actions.

- The first alternative option for the investor is self-investment. She does not choose that option if her net return on delegated investment, $Z(b_t^i)$, is larger than

$$1 + \rho. \tag{14}$$

- The second alternative option for the investor is to hire an old agent in period t and then hire a generation t manager at $t + 1$. At time t she would have to compensate the old agent enough to avoid shirking. This would entail promising the old agent compensation at least as large as $b_{t-1}^* R(b_{t-1}^*)$. Such compensation would attract the old manager irrespective of whether he is employed or not. Similarly, at time $t + 1$ the investor would have to pay the new recruit $b_t^* R(b_t^*)$. Hence, overall, if she were to opt for that deviation, the time t investor would expect to get

$$R(b_{t-1}^*)(1 - b_{t-1}^*) + \rho R(b_t^*)(1 - b_t^*) - \epsilon(1 + \rho), \tag{15}$$

where the last term ($\epsilon(1 + \rho)$) is the search cost of going after an old manager at t and then another one at $t + 1$.

- The third alternative option for the investor is to hire an old manager at t and then a young one at $t + 1$. In this case, when deviating, the generation t investor expects to pay $b_{t-1}^* R$ to

¹⁵This points to a benefit of agency rents: They compel the investor to compensate initial investment in skills by agents. This provides incentives for such investment, which, up to b^* , is socially optimal.

the old manager she hires at time t , and b_{t+1}^*R to the young manager she hires at time $t + 1$. Consequently, the deviating investor expects to earn

$$R(b_{t-1}^*)(1 - b_{t-1}^*) + \rho R(b_{t+1}^*)(1 - b_{t+1}^*) - \epsilon(1 + \rho). \quad (16)$$

- The fourth alternative option for the investor is to search for another young manager at time t , expecting to hire him for two periods and to compensate him with $\{w_t^{t*}, w_{t+1}^{t*}\}$ given in (11). In this case the investor expects to earn

$$Z(b_t^*) - \epsilon. \quad (17)$$

Overall, the employability constraint for the young manager is that $Z(b_t^*)$ be larger than or equal to (14), (15), (16), and (17). Since the expected gain of the young manager is increasing in b as long as he remains employable, his maximization problem is

$$\max_{b \in \Omega_t} b,$$

where Ω_t is

$$[b^*, b_{\max}] \cap \{b \text{ s.t. } Z(b) \geq \max[R(b_{t-1}^*)(1 - b_{t-1}^*) + \rho \max[R(b_t^*)(1 - b_t^*), R(b_{t+1}^*)(1 - b_{t+1}^*)] - \epsilon(1 + \rho), Z(b_t^*) - \epsilon]\}.$$

The solution of this maximization problem is stated in the next lemma.

Lemma 2: *The maximisation program of the young agent at time t has a unique solution b_t which is either equal to b_{\max} or such that*

$$Z(b_t) = \max[R(b_{t-1}^*)(1 - b_{t-1}^*) + \rho \max[R(b_t^*)(1 - b_t^*), R(b_{t+1}^*)(1 - b_{t+1}^*)] - \epsilon(1 + \rho), Z(b_t^*) - \epsilon]. \quad (18)$$

Equation (18) implicitly defines the function ϕ giving the solution of the maximisation problem at time t as a function of b_{t-1}^* , b_t^* and b_{t+1}^* , i.e., $b_t = \phi(b_{t-1}^*, b_t^*, b_{t+1}^*)$. That is, ϕ can be interpreted as a best response function. In equilibrium, the young investor must find it optimal to choose a level of complexity equal to b_t^* . Therefore, either $b_t^* = b_{\max}$ or

$$b_t^* = \phi(b_{t-1}^*, b_t^*, b_{t+1}^*). \quad (19)$$

It can never be the case that $Z(b_t^*) = Z(b_t^*) - \epsilon$, even when ϵ is arbitrarily small, as long as it is strictly positive. Thus the cost of searching for managers, even if it is very small, shuts down competition within the same generation.¹⁶ Consequently, (18) simplifies to

$$Z(b_t^*) = R(b_{t-1}^*)(1 - b_{t-1}^*) + \rho \max[R(b_t^*)(1 - b_t^*), R(b_{t+1}^*)(1 - b_{t+1}^*)] - \epsilon(1 + \rho). \quad (20)$$

When it is expected, that complexity will increase at $t + 1$, then either $b_t^* = b_{\max}$ (in which case $b_t^* \geq b_{t-1}^*$), or (20) simplifies to

$$R(b_t^*)(1 - b_t^*) - R(b_{t-1}^*)(1 - b_{t-1}^*) = -\beta b_t^* R(b_t^*) - \epsilon(1 + \rho). \quad (21)$$

$R(b)(1 - b)$ is the net return to an investor hiring a manager, with skill b , for one period. For $b \geq b^*$, this net return is decreasing with b , reflecting that the manager extracts an increasing fraction of the surplus.¹⁷ Since, the right-hand-side of (21) is negative, we have that $R(b_t^*)(1 - b_t^*) \leq R(b_{t-1}^*)(1 - b_{t-1}^*)$, that is $b_t^* \geq b_{t-1}^*$. Thus, between $t - 1$ and t , there is an increase in complexity, worsening agency problems, and eroding investors' returns while raising managers' rents. This is stated in the next lemma.

Lemma 3: *If $b_{t+1}^* \geq b_t^*$, then $b_t^* \geq b_{t-1}^*$.*

To interpret the increase stated in Lemma 3, consider the right-hand-side of (21). $\epsilon(1 + \rho)$, the cost incurred by investors searching for another manager, obviously limits competition between managers, in particular managers belonging to the same generation (as in Diamond, 1971). To focus on the specific economic mechanism at play in our model, which is driven by competition between managers from different generations, consider the limit case where ϵ goes to 0. In that case, the increase in b is solely driven by $\beta b_t^* R(b_t^*)$. This is the difference between the net investor's revenue when the principal hires an agent with type b_t^* on a long term basis ($Z(b_t^*)$) and when she hires the agent via

¹⁶As discussed below, this is similar to Diamond (1971), but, in contrast with Diamond (1971), in our model, managers from generation t also compete with their predecessors and successors.

¹⁷To see this note that $Z'(b)$, which for $b \geq b^*$ is negative, is equal to the derivative of $R(b)(1 - \delta b)$ plus a positive term.

a sequence of short-term contracts $((1 + \rho)R(b_t^*)(1 - b_t^*))$. This is a measure of the advantage of long term contracting, which is feasible with young agents, but not with old ones. Thus, it is a measure of the extent to which old managers are only imperfect substitutes for old ones. (21) shows how young managers take advantage of this imperfect substitutability: They raise complexity (and thus rents) above the prior level, up to the point at which investors are indifferent between i) hiring young managers on a long-term basis to manage more complex investment techniques, and ii) hiring old managers on a short-term basis to manage less complex techniques.

While Lemma 3 spells out what happens at time t when b is expected to rise after t , the next lemma states that future b s cannot decrease in equilibrium.

Lemma 4: b_t^* never decreases.

The intuition for Lemma 4 is the following. By Lemma 3, if b was to decrease at t , it would have to decrease also at $t+1$. Iterating, b would have to decrease below 0, which is a contradiction. Combining Lemmas 2, 3 and 4, we obtain our first proposition:

Proposition 1: *There exists a unique symmetric equilibrium. In that equilibrium, investors hire managers from their own generation for two periods. Equilibrium complexity, b_t^* , increases until it reaches b_{\max} . Starting from $b_0^* = b^*$, as long as $b_t^* < b_{\max}$, b_t^* is the unique solution of the recursive equation (21), which implicitly defines the function ψ mapping b_{t-1}^* into b_t^* .*

Proposition 1 directly implies the next corollary, which gives a lower bound on the increase in b_t^* due to imperfect competition among generations.

Corollary 1: *As long as $b_t^* < b_{\max}$, the growth of b_t^* is faster than exponential, i.e., starting from $b_0^* = b^*$,*

$$b_t^* \geq \frac{b^*}{(1 - \beta)^t}. \quad (22)$$

The greater the patience of the agent (β), the greater the advantage of long-term contracts over short term-contracts, the lower the substitutability among generations, the higher above b_{t-1}^* generation t can raise b_t^* . Hence the larger the lower bound on the growth of b_t^* .

Relation with Diamond (1971) and role of ϵ . If ϵ was strictly equal to 0, the equilibrium condition would be

$$Z(b_t^*) = \max[1 + \rho, R(b_{t-1}^*)(1 - b_{t-1}^*) + \rho \max[R(b_t^*)(1 - b_t^*), R(b_{t+1}^*)(1 - b_{t+1}^*)], Z(b_t^*)]. \quad (23)$$

The process b_t^* presented in Proposition 1 solves (23), and thus remains an equilibrium when $\epsilon = 0$. There are, however, other equilibria, in which the equilibrium value of b is between b^* (defined in (5)) and b_t^* (characterized in Proposition 1). In those equilibria, it is the last term (rather than the middle one) that binds in the max on the right-hand-side of (23). That is, the choice of b by a generation t manager is constrained by the choices of his competitors from the same generation (not by those of his predecessors).

(23) arises in the case where investors do not resample after drawing a manager. It is indeed weakly optimal for them to do so, and the above equilibrium relies on the best-response of managers to this weakly optimal strategy. Yet, when the search cost is strictly equal to 0 it is also weakly optimal for an investor to sample all the $1 - M$ managers that are not employed, after drawing a manager with b_t^* . If a manager anticipated such behaviour, then his best-response would be to opt for b_t^i slightly lower than b_t^* , to make sure he would eventually be drawn and hired. Since all managers would reason similarly, this would drive the equilibrium choice of b down to b^* . In this type of equilibrium, competition between managers would lead to the outcome preferred by investors.

These arguments, however, and the possibility for b^* to be an equilibrium, don't apply when ϵ is strictly positive, in which case the unique equilibrium is that characterized in Proposition 1. Thus ϵ , arbitrarily close to, but strictly above, 0, limits competition between managers belonging to the same generation. This is comparable to the way search costs limit competition and generate rents in Diamond (1971). One contribution of our analysis, relative to Diamond (1971), is to study the equilibrium dynamics of rents, and show they have a tendency to increase along the equilibrium path. To see this more clearly, note that the model in Diamond (1971) is similar to a one-period version of our model, where the equilibrium condition on the level of b prevailing at time 1 would be

$$Z(b_1^*) = \max[1 + \rho, Z(b_1^*) - \epsilon]. \quad (24)$$

(24) immediately leads to $b_1^* = b_{\max}$. This contrasts with our model where b_t^* progressively increases over several periods, before eventually reaching b_{\max} . The reason why the increase in b_t^* is only

progressive in our model is that the choice of generation t is constrained by the choices of previous generations. That anchor does not exist in Diamond (1971). Yet, in our model, the moderating effect of the previous generation is limited, due to imperfect substitutability between generations. Hence the gradual increase in b_t^* .

Externalities. When choosing b_{t-1}^* , generation $t - 1$ sets a benchmark, with which generation t will have to compete when choosing b_t^* . Thus, while the actions of generation $t - 1$ have no *exogenous direct* effect on the following generation, they exert an *endogenous externality* on the latter. When choosing a relatively high level of complexity b_{t-1}^* , generation $t - 1$ does not internalize that this will lead to an even larger level of complexity b_t^* , and thus large rents for generation t managers and low net returns for generation t investors.

Compensation and seniority. Lemma 1 implies that $w_t^t < w_{t+1}^t$. Thus, for a given generation, compensation rises with seniority, i.e., a given agent earns more when senior than when junior. Proposition 1 and Corollary 1, however, imply that senior managers from the previous generations earn less than junior managers from the current generation. Indeed, from Lemma 1, $w_t^{t-1*} \leq w_t^{t*}$ iff

$$b_{t-1}^* R(b_{t-1}^*) \leq (1 - \beta) b_t^* R(b_t^*). \quad (25)$$

Since $b_t^* \geq b_{t-1}^*$, (22) implies (25). This reflects that the increase in rents (driven by the increase in complexity) from one generation to the next is larger than the increase in compensation, within one generation, from one period to the next.

Example: In our simple example, where $R(b) = \min[\alpha b + 1, \bar{R}]$, the following corollary obtains:

Corollary 2: If $R(b) = \min[\alpha b + 1, \bar{R}]$ and (10) holds then as ϵ goes to 0, b_t^* goes to

$$\frac{b^*}{(1 - \beta)^t}. \quad (26)$$

In general, the increase in b_t^* above b^* , made possible by the imperfect substitutability between old and young managers, is enhanced by the fact that $R(b)$ increases in b . In the simple example,

however, $R(b)$ is constant when b is above b^* . In that situation, the increase in b_t^* is solely due to the imperfect substitutability between old and young managers. Correspondingly, the growth in b_t^* is just equal to its lower bound, stated in Corollary 1.

5 Welfare and Policy

For simplicity, we hereafter set $\rho = \beta$ and $b_{\max} = 1$.

5.1 Welfare

As discussed above, the optimal level of complexity for investors is b^* , which maximizes $Z(b)$, while for managers it is $b_{\max} = 1 \geq b^*$. Now turn to what a benevolent social planner would decide. Since utilities are linear, there is a unique Pareto optimum regarding real decisions, and the points on the Pareto frontier differ only in terms of purely redistributive transfers between investors and managers. In the first best, the social planner solves the following problem:

$$\max_{b \in [0,1]} W(b) = (1 + \beta)R(b) - Mcb.$$

The optimum is such that the marginal benefit of effort equals its marginal cost, i.e.,

$$b^{**} = R'^{-1}\left(\frac{Mc}{1 + \beta}\right).$$

Since $R'(1) = 0$, we have $b^{**} \leq 1$. Now,

$$W(b) = Z(b) + (bR(b) - Mcb).$$

Hence

$$\frac{\partial W(b)}{\partial b}\Big|_{b=b^*} = (R(b^*) - Mc) + b^*R'(b^*),$$

which, by (13), is positive. Consequently, $b^{**} \geq b^*$. We summarize this discussion in the next proposition:

Proposition 2: *The level of complexity preferred by investors is lower than the socially optimal level of complexity, which, in turn, is lower than the level of complexity preferred by the managers, i.e.,*

$$b^* \leq b^{**} \leq b_{\max}.$$

The fraction of total surplus obtained by investors is decreasing in b . Therefore they prefer b to be lower than the social optimum. In contrast, the fraction of total surplus obtained by managers is increasing in b , and therefore they want it to be higher than the social optimum.

5.2 Policy

In a “laissez faire” equilibrium, the level of complexity trends up from b^* to b_{\max} , which, as stated in Proposition 2, is above the social optimum. This raises the scope for policy intervention. To address this issue, we assume there is a regulator who can set simplicity and transparency standards, and monitor, at some cost, whether managers comply. To model this, we introduce an additional stage in the game. Each period t , at stage 0 the regulator announces a cap \underline{b}_t , and an inspection probability θ_t , to which we assume he can commit. Then, at stage 1, the young agent chooses b_t , and, with probability θ_t , is inspected by the regulator. In case of inspection, if $b_t > \underline{b}_t$ the agent is prevented from working. Otherwise, he enters the market and can meet an investor, and the game unfolds. Finally, we assume the monitoring technology is linear, i.e., there exists a constant $\gamma > 0$ such that the cost of monitoring with probability θ_t during one period is $\theta_t\gamma$. In this context we compare the performance of three policies:

- Laissez faire, in which the regulator never intervenes.
- Permanent monitoring, in which the regulator sets a constant cap \underline{b} , and monitors with constant probability θ to ensure that agents always comply with the regulatory cap.
- Periodic monitoring, which operates as follows: during $T - 1$ periods (with T finite and strictly larger than 1), laissez faire prevails, i.e., $\theta = 0$. Then, at the T^{th} period, the regulator intervenes, sets the maximum \underline{b} for this period, and monitors the agent with probability θ_T . Then a new cycle starts.

For simplicity, in this subsection, we focus on the simple example where $R(b) = \min[\alpha b + 1, \bar{R}]$ and (10) holds. In that case, W writes as

$$W(b) = [(1 + \beta)\alpha - Mc]b + (1 + \beta), \forall b \leq b^* = \frac{\bar{R}}{\alpha} \text{ and } W(b) = (1 + \beta)\bar{R} - Mcb, \forall b > b^*,$$

and we have the following lemma:

Lemma 5: *The level of complexity maximizing utilitarian welfare also maximizes investors' net returns, i.e., $b^{**} = b^*$.*

The lemma reflects the simple form of $R(b)$ in our example, where $R(b)$ is flat above b^* , so that there is no social gains from raising b above b^* . This enhances the tractability of our analysis of regulatory intervention.

5.2.1 Permanent monitoring

Suppose the regulator sticks to the stationary monitoring policy with constant standard, $\underline{b} \geq b^*$, and constant monitoring probability, θ . In contrast with the laissez faire environment, where equilibrium was unique, monitoring can generate multiple equilibria, with different levels of compliance. The intuition is the following. Under laissez faire, generation t managers are disciplined by their competitors from the previous generation. Thus, the equilibrium choice of b_t^* is uniquely determined by the predetermined level of b_{t-1}^* , i.e., $b_t^* = \psi(b_{t-1}^*)$, as stated in Proposition 1. In contrast, with regulation, as shown below, generation t managers are disciplined by their competitors from the same generation. Thus when he expects others to comply a time t , a generation t manager will tend to comply also, but when he expects the others not to comply, he will also tend not to comply.

To see this more formally, consider under what condition agent i would be employable at time t if he were to deviate to b_t^i when present and past generations are expected to comply:

$$Z(b_t^i) \geq \max[R(\underline{b})(1 - \underline{b}) + \rho R(\underline{b})(1 - \underline{b}) - \epsilon(1 + \beta), Z(\underline{b}) - \epsilon]. \quad (27)$$

The maximum on the right-hand side is $Z(\underline{b}) - \epsilon$.¹⁸ Thus, binding (27) yields $Z(b_t^i) = Z(\underline{b}) - \epsilon$. This gives the maximum deviation for which the manager is employable. In our simple example this is

$$b_t^i = \underline{b} + \frac{\epsilon}{R},$$

which goes to \underline{b} as ϵ goes to 0.

Now compare the expected gain of the agent who complies to that of the agent who deviates, but remains employable. The former is larger than the latter iff

$$\underline{b}(R(\underline{b}) - c) \geq (1 - \theta)b_t^i(R(b_t^i) - c). \quad (28)$$

¹⁸This contrasts with (18) where the maximum was not attained at $Z(b_t^*) - \epsilon$. This is why equilibrium is unique with laissez faire, not with regulation.

That is, in our simple case

$$\underline{b} \geq (1 - \theta)(\underline{b} + \frac{\epsilon}{R}).$$

A sufficient condition is

$$\theta \geq \frac{\epsilon}{R\underline{b}}.$$

Thus, when ϵ is very small, even if the probability of monitoring is very small, each manager prefers to comply if he expects others to. This is stated in the next proposition.

Proposition 3: *If market participants expect managers to comply, then when ϵ is very small, compliance is an equilibrium even if θ is very small.*

Proposition 3 emphasizes that if market participants expect compliance, this sets a benchmark inducing managers to comply. But what if market participants have the opposite expectation? For example, what happens if all previous generations have complied, but it is expected that, in spite of regulatory monitoring θ , all future market participants will deviate to the highest b_t^i at which they still are employable? In this context, it is easy to show (following the same logic as in the previous section) that the equilibrium b is larger for generation $t + 1$ than for generation t .¹⁹ Then, the highest feasible deviation at time t (anticipating that others deviate to \hat{b}) is b_t^i solving

$$Z(b_t^i) = R(\underline{b})(1 - \underline{b}) + \beta R(\hat{b})(1 - \hat{b}) - \epsilon(1 + \beta), \quad (29)$$

where the first term on the right-hand side of (29) is the net gain of the principal hiring an old agent at time t , while the second term is the net gain of the principal hiring an old agent at $t + 1$. In equilibrium, it must be that the value of b_t^i solving (29) is equal to \hat{b} . In our simple case, and setting $\epsilon = 0$, this pins down the candidate equilibrium value of \hat{b} at time t :

$$\hat{b} = \frac{\underline{b}}{1 - \beta}. \quad (30)$$

The other condition for a non-compliance equilibrium, is that managers prefer to deviate to \hat{b} rather than to comply with the regulatory cap \underline{b} . This is the case if (28) does not hold. In our simple case, this is

$$\theta \leq 1 - \frac{\underline{b}}{\hat{b}}. \quad (31)$$

¹⁹The intuition is that generation $t + 1$ faces easier terms, since it has less “virtuous” predecessors, than generation t .

Substituting (30) into (31) yields

$$\theta \leq \beta. \tag{32}$$

When managers don't comply, a fraction θ of them gets caught and cannot work. To avoid scarcity of managers, we assume

$$M \geq \frac{1}{1 - \beta}, \tag{33}$$

so that all principals can find and hire a manager. If (33) did not hold, managers' scarcity would be an additional reason for rents. To avoid introducing this new ingredient in the model, we focus on the case where (33) holds.

Thus, as long as $\theta \leq \beta$, while compliance is an equilibrium if all managers expect the others to comply, noncompliance is also an equilibrium if all managers expect the others not to comply. In particular, the laissez faire equilibrium path b_t^* characterized in Proposition 1 remains an equilibrium.

On the other hand, when $\theta > \beta$, whatever the beliefs of the managers about the behaviour of the others, all prefer to comply. We refer to that situation as “robust regulation.” The greater the patience β of the managers, the higher generation t can raise its b over that of the previous generation (as discussed after Corollary 1.) Hence the more tempting it is to deviate from \underline{b} , and the larger the monitoring frequency above which regulation is robust. Note also that the monitoring intensity in (32) does not depend on \underline{b} . Hence, the optimal choice under robust permanent regulation is to set $\underline{b} = b^*$. Correspondingly, when considering permanent monitoring, we hereafter set $\underline{b} = b^*$. The above discussion is summarized in the next proposition.

Proposition 4: *Suppose the regulator follows the permanent monitoring policy $(\theta, \underline{b} = b^*)$ and assume that ϵ goes to zero. Then, if $\theta > \beta$, there is a unique equilibrium, and in this equilibrium all agents comply. In contrast, if $\theta \leq \beta$ and (33) holds, each sequence $\{b_t\}_{t>0}$ such that $b_t \in \{b^*, \psi(b_{t-1})\}, \forall t > 0$, is an equilibrium.*

The equilibrium multiplicity in Proposition 4 highlights the complementarity arising in compliance environments, which can be interpreted in terms of social norms. When compliance is the norm, in the sense of expected behaviour, agents realize that, if they want to remain employable, they must comply

also. In contrast, when noncompliance is the norm, agents understand that they remain employable even if they don't comply, since their competitors also don't comply.

5.2.2 Periodic monitoring

We hereafter focus on robust regulation, i.e., regulation such that compliance is the only equilibrium. In this context, permanent regulation, which entails a constant monitoring probability of $\beta/(1 + \beta)$, is quite expensive if the cost of monitoring, γ , is large. Correspondingly, the utilitarian welfare in the laissez-faire steady-state

$$\bar{R}(1 + \beta) - Mcb_{\max}, \quad (34)$$

is larger than its counterpart under permanent robust monitoring

$$\bar{R}(1 + \beta) - Mcb^* - \frac{\beta}{1 + \beta}\gamma, \quad (35)$$

if

$$\gamma > \frac{1 + \beta}{\beta}Mc(b_{\max} - b^*). \quad (36)$$

Yet, even when (36) holds, it can be optimal to monitor, but only infrequently. Our next proposition states that for intermediary values of the monitoring cost γ , periodic monitoring dominates both laissez-faire and permanent monitoring.

Proposition 5:

If $b_{\max} \geq (1 + \beta)b^$, there exist thresholds $0 < \gamma_1 < \gamma_2$ such that, if $\gamma_1 < \gamma < \gamma_2$, periodic interventions, at intervals of two or more periods, are better than laissez-faire and constant monitoring.*

The economic intuition behind Proposition 5 combines two insights already analyzed in the paper. First, new generations cannot deviate too much from elder generations in their choice of b . So there is some form of disciplining effect from elder generations on new generations. To save on monitoring costs, the regulator might rely for some periods on this disciplining effect, and then intervene only after b has risen a lot. Second, monitoring is only effective if a critical mass of the population is inspected. Similarly to fixed costs, this makes infinitesimal levels of monitoring suboptimal. Thus it is better to rarely monitor a lot than to always monitor a little bit. This is not unlike the dynamics

arising for optimal investment with fixed costs. The technical condition $b_{\max} \geq (1 + \beta)b^*$ allows that there is enough space for b_t to move according to its laissez-faire dynamics.

5.3 Normative and positive implications

Our theoretical model implies that it can be optimal to have periods of intense monitoring where the regulator performs a "crack-down" that resets rents at a low level, followed by several periods of lighter monitoring. This implies that monitoring expenses can be cyclical and should be allowed to experience spikes: it is optimal to grant regulators with abnormally high levels of funding from time to time, so that they can hire human capital to perform exceptional amounts of monitoring. In other words, the usual flow cost of supervision is optimally smaller than what is spent in special times. The governance and mandate of regulators should thus allow them to expand their budget temporarily when a "reset" is needed.

Our analysis of monitoring cycles also has empirical implications. In our model, regulatory changes are slowly offset by agents, who bring back opacity and rents to their initial level. This means that the short-term and long-term effects of regulations should be different. This potentially matters for empirical studies using regulatory shocks as natural experiments: the short-term impact of a reform should be expected to be higher than its long-term impact. In our model, the speed at which the offsetting of regulation takes place is endogenous. Our model predicts a progressive build-up in rents and complexity following deregulation episodes, in line with empirical results from Philippon and Resheff (2008).

6 Conclusion

We analyze the dynamics of agency rents. Our key result is that, in equilibrium, transparency and simplicity standards progressively deteriorate, while rents rise.

In our model, successive generations of agents choose their skills, determining their productivity, as well as the severity of their agency problems. Because of the link between incentives and horizons, young and old generations are not perfect substitutes. Thus, young agents can choose more rent-extracting technologies than their elder peers. They, however, are still constrained not to deviate too

much from the choices of older generations. This leads to a progressive increase in the rents extracted by agents, and decrease in the principal's surplus.

We believe this model is well suited to understand the slow moving rise of rents in finance, an industry characterized by a high level of technological flexibility. Over time, agents choose more complex and opaque investment techniques that might be more productive in gross terms, but also increase rents, so that investors ultimately lose from such technological trend. In the context of our model, regulatory intervention can be welfare improving. Because older generations constrain (albeit imperfectly) the choice of future generations, permanent regulation can be dominated by cyclical regulation, where transparency and simplicity standards are periodically reset at a low level. Between two regulatory crack-downs, rents and opacity drift upward.

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Proofs:

Proof of Lemma 1: The Lagrangian is

$$\mathcal{L} = R(1 + \rho) - w_t^t - \rho w_{t+1}^t + \mu_t(w_t^t + \beta w_{t+1}^t - b_t R) + \mu_{t+1}(w_{t+1}^t - b_t R),$$

where μ_t and μ_{t+1} are the multipliers of the time t and $t+1$ incentive constraints, respectively. The first order condition with respect to w_t is: $-1 + \mu_t = 0$. Hence the incentive compatibility constraint at time t binds, i.e., $w_t^t + \beta w_{t+1}^t = b_t R$. The first order condition with respect to w_{t+1} is: $-\rho + \mu_t \beta + \mu_{t+1} = 0$. Substituting $\mu_t = 1$, $\mu_{t+1} = \rho - \beta$. When $\rho > \beta$, $\mu_{t+1} > 0$, so that the the incentive compatibility constraint at time $t + 1$ also binds. Hence, the optimal compensation is as stated in the lemma.

QED

Proof that, under (9), shirking once is dominated by effort at both periods: Effort at both periods dominates shirking at t , followed by effort at $t + 1$, if

$$R(b_t)[(1 + \rho) - (1 + \rho - \beta)b_t] \geq R(b_t)[(1 - \Delta) + \rho(1 - b_t)].$$

That is

$$\frac{\Delta}{1 - \beta} \geq b_t. \tag{37}$$

Effort at both periods dominates effort at t , followed by shirking at $t + 1$, if

$$R(b_t)[(1 + \rho) - (1 + \rho - \beta)b_t] \geq R(b_t)[(1 - b_t) + \rho(1 - \Delta)].$$

That is

$$\frac{\rho \Delta}{\rho - \beta} \geq b_t. \tag{38}$$

Now

$$\frac{1 + \rho}{1 + \rho - \beta} < \frac{1}{1 - \beta} \leq \frac{\rho}{\rho - \beta},$$

Hence if (8) holds (which it does under (9)), then (37) and (38) also hold, so that effort at both periods is optimal.

QED

R and Z in the simple example: If $R(b) = \min[\alpha b + 1, \bar{R}]$,

$$Z(b) = (\alpha b + 1)[(1 + \rho) - (1 + \rho - \beta)b], \forall b \leq \frac{\bar{R} - 1}{\alpha} \text{ and } Z(b) = \bar{R}[(1 + \rho) - (1 + \rho - \beta)b], \forall b > \frac{\bar{R} - 1}{\alpha}.$$

Thus

$$Z'(b) = \alpha[(1 + \rho) - 2(1 + \rho - \beta)b] - (1 + \rho - \beta), \forall b \leq \frac{\bar{R} - 1}{\alpha} \text{ and } Z'(b) = -(1 + \rho - \beta)\bar{R}, \forall b > \frac{\bar{R} - 1}{\alpha}.$$

Thus, $Z' \leq 0$ for $b \geq \frac{\bar{R} - 1}{\alpha}$. For $b \leq \frac{\bar{R} - 1}{\alpha}$, $Z' \geq 0$ if and only if

$$\frac{\alpha(1 + \rho) - (1 + \rho - \beta)}{2(1 + \rho - \beta)} \geq \alpha b.$$

This holds for all $b \leq \frac{\bar{R} - 1}{\alpha}$, if

$$\frac{\alpha(1 + \rho) - (1 + \rho - \beta)}{2(1 + \rho - \beta)} \geq \bar{R} - 1.$$

That is

$$\frac{1 + \rho}{(1 + \rho - \beta)} \geq \frac{2\bar{R} - 1}{\alpha},$$

i.e., (10) holds. Hence, (10) implies $b^* = \frac{\bar{R} - 1}{\alpha}$.

Proof of Lemma 2: Ω is non-empty because $b^* \in \Omega$. Indeed

$$Z(b^*) \geq \max[R(b_{t-1}^*)(1 - b_{t-1}^*) + \rho \max[R(b^*)(1 - b^*), \rho R(b_{t+1}^*)(1 - b_{t+1}^*)] - \epsilon(1 + \rho), Z(b_t^*) - \epsilon],$$

since $Z(b)$ is decreasing in b for $b \geq b^*$. Ω is compact. This compact subset of the real line has a unique maximum, b_t , which defines the unique solution of the maximisation program of the agent.

If $b_t \neq b_{max}$, it must be that $b_t < b_{max}$. If (18) did not hold, this would imply that the left-hand side of (18) would be strictly above its right-hand side. This strict inequality would by continuity extend to a neighbourhood of b_t included in $[0, b_{max}]$, which would contradict the fact that b_t is the maximum of Ω . So, either $b_t = b_{max}$, or b_t solves (18).

QED

Proof of Lemma 4: By Lemma 3, if b is to decrease between $t - 1$ and t , i.e., $b_t^* < b_{t-1}^*$, we must have $b_{t+1}^* < b_t^*$. Then, as long as $b \leq b_{max}$, (20) is

$$R(b_t^*)(1 + \rho)(1 - b_t^*) + \beta b_t^* R(b_t^*) = R(b_{t-1}^*)(1 - b_{t-1}^*) + \rho R(b_{t+1}^*)(1 - b_{t+1}^*) - \epsilon(1 + \rho). \quad (39)$$

Denote $g(b_t) = R(b_t)(1 - b_t)$. In terms of g , (39) writes as

$$g(b_t^*)(1 + \rho) + \beta b_t^* R(b_t^*) = g(b_{t-1}^*) + \rho g(b_{t+1}^*) - \epsilon(1 + \rho).$$

That is

$$g(b_t^*) = \frac{g(b_{t-1}^*) + \rho g(b_{t+1}^*)}{1 + \rho} - \frac{\beta b_t^* R(b_t^*)}{1 + \rho} - \epsilon(1 + \rho). \quad (40)$$

Since

$$\frac{g(b_{t-1}^*) + \rho g(b_{t+1}^*)}{1 + \rho} - \frac{\beta b_t^* R(b_t^*)}{1 + \rho} - \epsilon(1 + \rho) < \frac{g(b_{t-1}^*) + \rho g(b_{t+1}^*)}{1 + \rho},$$

we have

$$g(b_t^*) < \frac{g(b_{t-1}^*) + \rho g(b_{t+1}^*)}{1 + \rho}.$$

By Jensen inequality (as g is concave and decreasing), this implies

$$b_t^* > \frac{b_{t-1}^* + \rho b_{t+1}^*}{1 + \rho},$$

that is

$$b_t^* - b_{t+1}^* > \frac{1}{\rho}(b_{t-1}^* - b_t^*).$$

Because $\frac{1}{\rho} > 1$, This implies that, as t goes to infinity, $b_t^* - b_{t+1}^*$ goes to plus infinity, which, since $b_t^* \leq b_{\max}$, implies b_{t+1}^* goes to minus infinity, a contradiction since $b \geq 0$.

QED

Proof of Corollary 1: (21) rewrites as

$$\frac{R(b_t^*)}{R(b_{t-1}^*)} = \left(\frac{1 - b_{t-1}^*}{1 - (1 - \beta)b_t^*} \right) - \frac{\epsilon(1 + \rho)}{[1 - (1 - \beta)b_t^*]R(b_{t-1}^*)}. \quad (41)$$

Proposition 1 implies that the left-hand side of (41) is larger than one. Hence (41) implies

$$\left(\frac{1 - b_{t-1}^*}{1 - (1 - \beta)b_t^*} \right) - \frac{\epsilon(1 + \rho)}{[1 - (1 - \beta)b_t^*]R(b_{t-1}^*)} \geq 1,$$

which, in turn yields (22).

QED

Proof of Corollary 2: In the simple example, for $b \geq b^*$, (21) simplifies to

$$b_t^* = \frac{b_{t-1}^*}{1-\beta} + \frac{\epsilon(1+\rho)}{(1-\beta)\bar{R}}.$$

Thus, as ϵ goes to 0, we get (26).

QED

Proof of Lemma 5: $W(b)$ decreases with b , $\forall b > b^*$. $\forall b \leq b^*$, $W(b)$ increases with b if $(1+\beta)\alpha \geq Mc$. This is implied by our assumption that $R(b^*) \geq Mc$, if

$$(1+\beta)\alpha \geq R(b^*) = \bar{R}.$$

Now, since $\beta = \rho$, condition (10) simplifies to

$$\alpha(1+\beta) \geq 2\bar{R} - 1.$$

Hence, $W(b)$ increases with b if

$$\bar{R} \geq 1,$$

which holds. Hence, $W(b)$ increases with b , and $b^{**} = b^*$.

QED

Proof of Proposition 5:

To prove Proposition 5, we first characterize the equilibrium dynamics of b_{kT+t} , over a cyclical intervention regime where the regulator resets b to $\underline{b} \geq b^*$ every T periods (we will conditions under which such cycle dominates both laissez-faire and permanent monitoring). We then study the condition under which the agent complies after T periods. Finally we compare $W(1)$ (welfare under permanent monitoring) and $W(\infty)$ (welfare under laissez-faire) to $W(T)$, for T finite and larger than 2. A series of technical lemmas are needed to complete the proof. We directly assume $\epsilon \rightarrow 0$ in formulas and reasoning where ϵ plays no role but explicitly introduce ϵ at intermediary steps where it plays a role in the determination of the equilibrium.

Lemma 6: Consider a policy that resets b to \underline{b} with periodicity T . The dynamics of b_t is described by:

$$b_{kT+n} = \min\left(\frac{b_{kT+n-1}}{1-\beta}, b_{\max}\right) = \min\left(\frac{\underline{b}}{(1-\beta)^n}, b_{\max}\right), \forall n \in \{1, \dots, T-2\}, \text{ if } T > 2,$$

while

$$b_{(k+1)T-1} = \min(b_{(k+1)T-2} + \beta \underline{b}, b_{\max}) = \min\left(\left(\frac{1}{(1-\beta)^{T-2}} + \beta\right)\underline{b}, b_{\max}\right).$$

Proof of Lemma 6:

Assume for now that b_{\max} is not reached during the cycle. First consider the case $T > 2$. At time $t = kT + n$, where $n \in \{1, \dots, T-2\}$, no monitoring happens, so that the analysis is similar to our benchmark case: a generic young manager from generation t chooses the highest possible b_t^i at which she is still employable. Assume this does not bind b_{\max} . Following the same reasoning as in the previous analyses, binding the employability constraint, yields

$$\bar{R}(1 + \beta) - b_t \bar{R} = \bar{R}(1 + \beta) - b_{t-1} \bar{R} - \beta \min[b_t, b_{t+1}] \bar{R},$$

which, simplifies to

$$\bar{R}(1 + \beta) - b_t \bar{R} = \bar{R}(1 + \beta) - b_{t-1} \bar{R} - \beta b_t \bar{R}.$$

Thus

$$b_t = \frac{b_{t-1}}{1 - \beta}.$$

This implies

$$b_{kT+n} = \frac{b_{kT+n-1}}{1 - \beta}, \forall n \in \{1, \dots, T-2\}.$$

For the last generation before regulatory intervention, $t = (k+1)T - 1$, things are slightly different since a young manager anticipates that $b_{t+1} = \underline{b}$. Hence, given his expectation b_t on the behavior of his contemporaneous peers, a manager i in that generation chooses b_t^i such that

$$\bar{R}(1 + \beta) - b_t^i \bar{R} = \max[\bar{R}(1 + \beta) - b_{t-1} \bar{R} - \beta \underline{b} \bar{R} - (1 + \rho)\epsilon, \bar{R}(1 + \beta) - b_t \bar{R} - \epsilon].$$

$$b_t^i = \min[b_{t-1} + \beta \underline{b} + \rho\epsilon/\bar{R}, b_t] + \epsilon/\bar{R}.$$

In equilibrium, $b_t^i = b_t$, which prevents b_t to be the argmin in the expression above at equilibrium, and thus (applying $\epsilon \rightarrow 0$):

$$b_t = b_{t-1} + \beta \underline{b}.$$

Since $t - 1 = kT + T - 2$, $b_{t-1} = \frac{\underline{b}}{(1-\beta)^{T-2}}$. Thus we have

$$b_t = \left(\frac{1}{(1-\beta)^{T-2}} + \beta \right) \underline{b}. \quad (42)$$

Second, consider the case $T = 2$. At $t = 1$, we have $b_1 = \underline{b}$, at $t = 2$, we have $b_2 > \underline{b}$, ... b_2 binds the employability constraint:

$$b_2 = \min[b_1 + \beta \underline{b}, b_2] + (1 + \beta)\epsilon / \bar{R}.$$

Hence (applying $\epsilon \rightarrow 0$):

$$b_2 = (1 + \beta)\underline{b}, \quad (43)$$

which is consistent with (42) evaluated at $T = 2$.

In the lemma, we allow for the possibility that b_{\max} is reached during the cycle. This means that $b_{kT+n} \leq b_{\max}$ is also part of the employability constraints that can bind, hence the min in the formulas.

QED

Now, turn to the monitoring probability ensuring compliance. At the time of intervention, $t = kT$, the regulator sets the desired level of complexity below the previous level, i.e., $\underline{b} < b_{t-1}$. Consider a generic agent i from generation t . If he complies, he gets

$$\underline{b}\bar{R}.$$

If he deviates, he chooses the maximum level of complexity at which he is still employed, i.e., b_t^i such that

$$\bar{R}(1 + \beta) - b_t^i \bar{R} = \max\{\bar{R}(1 + \beta) - b_{t-1}^* \bar{R} - \beta \min[b_t, b_{t+1}] \bar{R} - (1 + \beta)\epsilon, \bar{R}(1 + \beta) - b_t \bar{R} - \epsilon\}. \quad (44)$$

Where b_t are the expectations of agent i about the behavior of other members of his generation and b_{t+1} his expectations on the behavior of the next generation. b_{t-1}^* is the equilibrium level of opacity of the generation just prior to monitoring time, which we already computed. We look for an

intensity of monitoring that implements "robust compliance", i.e. we want compliance to happen even if agents expect that their contemporaneous and future peers deviate to the highest feasible deviation b_t compatible with optimisation. This highest (collective) deviation b_t is given for generation t by the equation:

$$\bar{R}(1 + \beta) - b_t \bar{R} = \bar{R}(1 + \beta) - b_{t-1} \bar{R} - \beta \min[b_t, b_{t+1}] \bar{R} - (1 + \beta)\epsilon. \quad (45)$$

This is because if this equation did not hold, then the individual deviation b_t^i defined above would be determined by the comparison to current generations, $\bar{R}(1 + \beta) - \hat{b} \bar{R} = \bar{R}(1 + \beta) - b_t \bar{R} - \epsilon$, such that $b_t^i > b_t$, which would allow us to consider a strictly higher rational deviation of the rest of generation t , namely $b_t = b_t^i$, in contradiction with the definition of b_t as the highest feasible deviation. Moreover, since generation $t + 1$ is not monitored and since we assume future generations that are monitored do not comply, we have $b_t \leq b_{t+1}$.

Now, the definition of the highest (collective) feasible deviation b_t simplifies to

$$b_t = \frac{b_{t-1}^*}{1 - \beta}. \quad (46)$$

Looking for robust monitoring, the regulator will choose the lowest possible level of monitoring ensuring compliance, so that an equilibrium where agents all start to deviate at some point is excluded. Equation (28) pins down θ as the solution of:

$$\underline{b} = (1 - \theta) \left(\frac{b_{t-1}^*}{1 - \beta} \right)$$

Lemma 7: for a given \underline{b} , longer cycles imply weakly higher monitoring expense at the time of reset, i.e. $\theta(T) \leq \theta(T + 1)$.

Proof of Lemma 7:

Noting $b_{kT+i}(T)$ the equilibrium path on a T -cycle of length T and reset at \underline{b} , from Lemma 5, we know that $b_{kT-1}(T) \leq b_{k(T+1)-1}(T + 1)$, from which we conclude that $\theta(T) \leq \theta(T + 1)$.

QED

This allows us to now characterize further policies that cannot be optimal in the set of cyclical policies. This is useful later in establishing proposition 3.

Lemma 8: A periodic robust monitoring policy of period T that is optimal among cyclical policies (including laissez-faire) must be such that $b_{(k+1)T-1} < b_{\max}$.

Proof of Lemma 8:

Consider a periodic robust monitoring policy such that $b_{T-1} = b_{\max}$ and $b_T = \underline{b}$. We are going to show that it is dominated by either laissez faire or by the periodic monitoring policy with period $T - 1$.

Either $W(T) \leq W(\infty)$, and the lemma is true, or $W(T) > W(\infty)$. Consider the latter case. We want to establish that $W(T - 1) > W(T)$. We can explicit welfare over the cycle of length T with reset at \underline{b} as follows: $W(T) = \bar{R}(1 + \beta) - cMb_{average}(T) - \frac{\gamma}{T}\theta(T)$ where $b_{average}(T)$ is the average of b over the cycle. The $(T - 1)$ -cycle with starting point \underline{b} has an identical dynamics of b as the cycle of length T along the first $(T - 2)$ periods, a weakly lower b at period $T - 1$ (a consequence of lemma 5) and a smaller cost of monitoring ($\theta(T - 1) \leq \theta(T)$ from Lemma 6). Thus, we have $b_{average}(T - 1) \leq b_{average}(T)$, such that: $W(T) \leq \frac{T-1}{T}W(T - 1) + \frac{1}{T}W(\infty)$. This implies $W(T - 1) > W(T)$ because we know that $W(T) > W(\infty)$.

QED

To finally estimate welfare over cycles of various lengths, we need to compute the monitoring costs associated to them. This is what the following lemma does:

Lemma 9: To ensure compliance with periodic robust monitoring, under an optimal periodic monitoring cycle of period T , the regulator sets the monitoring probability to:

$$\theta_T = 1 - \frac{1 - \beta}{\left(\frac{1}{(1-\beta)^{T-2}} + \beta\right)}.$$

Proof of Lemma 9:

From lemma 7, we know that we can consider that b_{\max} is not reached during the cycle.

$$\underline{b} = (1 - \theta)\left(\frac{b_{t-1}^*}{1 - \beta}\right) = \frac{1 - \theta}{1 - \beta}\left(\frac{1}{(1 - \beta)^{T-2}} + \beta\right)\underline{b}$$

$$\theta = 1 - \frac{1 - \beta}{\left(\frac{1}{(1-\beta)^{T-2}} + \beta\right)}$$

Thus, for $T = 2$, we have

$$\theta = \frac{2\beta}{1 + \beta}.$$

Thus, whether $T = 2$ or $T > 2$, the monitoring cost is independent of the chosen reset level \underline{b} .

QED

We can now compute welfare over a cycle that does not reach b_{\max} , which yields the following lemma:

Lemma 10: Average welfare under a periodic robust monitoring cycle of period $T = 2$ resetting b to \underline{b} , is:

$$\bar{R}(1 + \beta) - \frac{1 + (1 + \beta)}{2} M c \underline{b} - \frac{1}{2} \left(\frac{2\beta}{1 + \beta} \right) \gamma. \quad (47)$$

Proof of Lemma 10:

Given $T = 2$, we compute welfare over two periods where b_t oscillates between the two values \underline{b} and $(1 + \beta)\underline{b}$.

QED

We now compare welfare under periodic robust monitoring to its counterpart under permanent monitoring and under laissez faire. The next proposition states a preliminary result.

Lemma 11: Assume $b_{\max} \geq (1 + \beta)b^*$. Welfare under monitoring with $T = 2$ and reset at b^* is larger than welfare under laissez faire and than welfare under permanent monitoring iff:

$$\frac{1 + \beta}{\beta} M c b^* < \gamma < \frac{1 + \beta}{\beta} M c (b_{\max} - (1 + \frac{\beta}{2})b^*).$$

Proof of Lemma 11:

(47) is greater than (34), iff

$$\frac{1 + (1 + \beta)}{2} M c b^* + \frac{\beta}{1 + \beta} \gamma < M c b_{\max}. \quad (48)$$

That is

$$\gamma < \frac{1 + \beta}{\beta} M c (b_{\max} - (1 + \frac{\beta}{2})b^*).$$

Note that the right-hand-side of is positive, since $b_{\max} \geq (1 + \beta)b^*$.

(47) is greater than (35), iff

$$\bar{R}(1 + \beta) - \left(\frac{1 + (1 + \beta)}{2} M c b^* + \frac{\beta}{1 + \beta} \gamma \right) > \bar{R}(1 + \beta) - (c M b^* + \beta \gamma)$$

that is:

$$\gamma > \frac{1+\beta}{\beta} M c b^*$$

This completes the proof of proposition 5, by choosing $\gamma_1 = \frac{1+\beta}{\beta} M c b^*$ and $\gamma_2 = \frac{1+\beta}{\beta} M c (b_{\max} - (1 + \frac{\beta}{2})b^*)$, a 2-cycle being a particular case among cycles. Indeed, $\gamma_1 < \gamma_2$ as soon as

$$b_{\max}/b^* > (1 + \frac{\beta}{2})$$

which is implied by our assumption $b_{\max} \geq (1 + \beta)b^*$.

QED

Figure 1: time line during period t for generation born at time t

Stage 1:	Stage 2:	Stage 3:	Stage 4:
Each generation t agent chooses b_t^i	Each generation t investor matched with an agent	Agent accepts or not	Agent exerts effort or not
	Investor observes his agent's b_t^i	Investor searches or not	Output realized
	Investor makes take it or leave it offer	Investment takes place	Compensation and consumption take place

Figure 2: A simple example

